Demonstration of Linear Digital Filters (FIR)

A. Linear digital filters may be conceived of as vectors of weights that are to be multiplied by the digitally sampled values from a waveform. The filters given below are both 11 point digital filters with a half-amplitude frequency cutoff of approximately 17.5 Hz for data sampled at 200 Hz.

LOW PASS FILTER COEFFICIENT	LAG	HIGH PASS FILTER COEFFICIENT	LAG
0.0166	5	-0.0166	5
0.0402	4	-0.0402	4
0.0799	3	-0.0799	3
0.1231	2	-0.1231	2
0.1561	1	-0.1561	1
0.1684	0	0.8316	0
0.1561	-1	-0.1561	-1
0.1231	-2	-0.1231	-2
0.0799	-3	-0.0799	-3
0.0402	-4	-0.0402	-4
0.0166	-5	-0.0166	-5

So, what does all that fancy jargon mean?

1. The fact that these are 11 point filters indicates that 11 sample points are used in the determination of the new filtered value of any one sample point. Specifically, the sixth sample point is a weighted sum of the first 11 samples. In general, for an 11 point filter, any filtered sample point is the weighted sum of the preceding 5 sample values, the unfiltered sample value itself, and the subsequent 5 sample values. The lag refers to which sample point, relative to the center sample point, is to be weighted by the particular coefficient.
The non-recursive filter uses raw sample values in the calculations; recursive filters use the already filtered values of preceding samples in the calculations. Non-recursive filters are more straightforward and more commonly used.

2. The term linear denotes that the filter involves the computation of weighted sums of the digital sample values. Other filtering algorithms can be devised, but are not often applied to psychophysiological signals.

3. Digital filters, unlike analog filters, have characteristics that are sampling-rate dependent. These same filters would have a different cutoff frequency for data sampled at different sampling rates. Once you know the characteristics of a digital filter at a given frequency, it is a simple matter to convert the filter to another sampling rate as follows:

17.5/200 = x/1000 ; x = 87.5 @ 1000 Hz Sampling rate

17.5/200 = x/20 ; x = 1.75 @ 20 Hz Sampling rate

B. So, you may ask, how do I work these things? First, consider a much simpler filter with the coefficients [.25 .50 .25]. This is a three point low pass filter. Consider the digitally sampled wave:

[0 2 6 4 10 4 6 2 0 -2 -6 -4 -10 -4 -6 -2 0]

You should plot out these sample values as a function of time to examine the waveform.

1. The filter is applied as follows: the three filter coefficients are multiplied by the first three sample points and then summed to yield the middle sample point. So, .25*0 + .5*2 + .25*6 = 2.5 ; 2.5 is thus the new value for the second sample point. The filter "window" is then "shifted" or "slid" to the next 3 sample points and the procedure is repeated. Applying the filter to the entire sampled waveform yields:

[-- 2.5 4.5 6.0 7.0 6.0 4.5 2.5 0 -2.5 -4.5 -6.0 -7.0 -6.0 -4.5 -2.5 --]

Now, **you should plot out these filtered sample values on the same graph** you just constructed to examine the effects of filtering. Notice how the waveform has been smoothed.

2. Notice also that there exist no filtered values for the first and last samples. For most linear digital filters, some sample loss occurs. In general, the number of samples lost is P - 1, where P is the number of points in the filter; (P-1)/2 samples are lost at the beginning of the epoch and (P-1)/2 samples are lost at the end of the epoch.

If you like metaphors, this is the digital filter's "capacitor" which needs to charge. More precisely, the filter needs to take into account the trend or changes in the waveform; the discontinuities at the beginning and end of the epoch make such a trend analysis impossible. The most straightforward solution is therefore to refrain from filtering the earliest and latest samples in each epoch since the information required is unavailable.

3. **You should apply this simple high-pass filter [-.25 .50 -.25] to the original waveform to see what happens. Plot the results**.

C. The more elaborate 11 point filters are applied in the same manner. This will be demonstrated using basic matrix algebra. If the original waveform is represented by the matrix:

$$\mathbf{M} = [t_0, t_1, t_2, t_3, \ldots, t_{n-1}]_{1xn}$$

then the filtered matrix (where f_0 is actually at time point ((P-1)/2)+1):

$$\mathbf{F} = [f_0, f_1, f_2, f_3, \ldots, f_{m-1}]_{1xm} \quad \text{(where } m = n - (P-1))$$

is obtained by vector multiplication as follows:

1. First a matrix of shifted sample values is created:

$$
\mathbf{S} = [t_0, t_1, t_2, t_3, \ldots, t_{10} \\
t_1, t_2, t_3, t_4, \ldots, t_{11} \\
t_2, t_3, t_4, t_5, \ldots, t_{12} \\
\ldots \\
t_{n-11}, t_{n-10}, \ldots, t_{n-1}]_{mxP}
$$

2. This matrix is then post-multiplied by the filter coefficient matrix $\mathbf{C}_{Px1}$ (**C** is either of the 11 point filter vectors listed at the top of the handout) to yield the filtered matrix of sample values:

$$\mathbf{F'}_{mx1} = \mathbf{S}_{mxP} * \mathbf{C}_{Px1}$$

3. The following pages of the handout contain plots of data filtered by the above procedure. First, the raw data are presented. Three waveforms were generated for a 20 Hz sampling rate (what's the Nyquist frequency?). The .5 Hz and 2 Hz waveforms were simply added together in phase to produce the more complex waveform which is to be filtered. The last two columns represent the high-pass and low-pass filtered versions of this more complex waveform. At a 20 Hz sampling rate, the 1/2-amplitude frequency cutoff of these filters is 1.75 Hz. Because the roll-off characteristics of these filters are rather broad, the original .5 & 2 Hz waveforms were not reproduced with fidelity. Nonetheless, you can see that fairly close approximations are produced. Plots of the original three waveforms and of the filtered versions follow.

4. In general, two features are important in determining a filter's length: roll off and 1/2 amplitude frequency. The sharper the roll-off desired, the longer the filter must be. The lower the 1/2 amplitude frequency, again the longer the filter. This last relationship should be intuitive: longer frequencies require more sample points to capture 1 wave period.

The last thing you should remember, now that you have the hang of digital filtering: digital filters are not always intuitive!

Generated waveforms: 1 Hz & 4 Hz Sine waves and linear combination
Filtered versions of the linear combination

TIME (msec)	.5 Hz	2 Hz	SUM	------Filtered------ High Pass	Low Pass
0	0.0	0.0	0.0	---	---
50	.156	.588	.744	---	---
100	.309	.951	1.260	---	---
150	.454	.951	1.405	---	---
200	.588	.588	1.176	---	---
250	.707	.000	.707	.04103	.66596
300	.809	-.588	.221	-.33555	.55655
350	.891	-.951	-.060	-.56681	.50681
400	.951	-.951	.000	-.56335	.56335
450	.988	-.588	.400	-.32477	.72477
500	1.000	-.000	1.000	.05826	.94173
550	.988	.588	1.575	.43948	1.1355
600	.951	.951	1.902	.67404	1.2280
650	.891	.951	1.842	.67058	1.1714
700	.809	.588	1.397	.42970	.96730
750	.707	.000	.707	.04103	.66596
800	.588	-.588	.000	-.34822	.34822
850	.454	-.951	-.497	-.59229	.09529
900	.309	-.951	-.642	-.60075	-.04125
950	.156	-.588	-.431	-.37297	-.05803
1000	.000	-.000	-.000	-.00000	.00000
1050	-.156	.588	.431	.37297	.05803
1100	-.309	.951	.642	.60075	.04125
1150	-.454	.951	.497	.59229	-.09529
1200	-.588	.588	.000	.34822	-.34822
1250	-.707	.000	-.707	-.04103	-.66596
1300	-.809	-.588	-1.397	-.42970	-.96730
1350	-.891	-.951	-1.842	-.67058	-1.1714
1400	-.951	-.951	-1.902	-.67404	-1.2280
1450	-.988	-.588	-1.575	-.43948	-1.1355
1500	-1.000	-.000	-1.000	-.05826	-.94173
1550	-.988	.588	-.400	.32477	-.72477
1600	-.951	.951	-.000	.56335	-.56335
1650	-.891	.951	.060	.56681	-.50681
1700	-.809	.588	-.221	.33555	-.55655
1750	-.707	.000	-.707	---	---
1800	-.588	-.588	-1.176	---	---
1850	-.454	-.951	-1.405	---	---
1900	-.309	-.951	-1.260	---	---
1950	-.156	-.588	-.744	---	---
2000	-.000	-.000	-.000	---	---

P.S. : Answer to High Pass Filter Excercise

[-- -.5 1.5 -2 3 -2 1.5 -.5 0 .5 -1.5 2 -3 2 -1.5 .5 --]